

Principles of Economics and Management I

Ciências ULisboa · MMAEG · 2026/2027

Problem Set 6 · Uncertainty, Risk and Information

Session 6 · LN06

Exercise level: **[Easy]** routine / warm-up **[Medium]** typical test level **[Hard]** challenge (above test level, to consolidate mastery)

1. (2 points) **[Easy] Expected value and expected utility.** A gamble pays €10,000 with probability $1/2$ and €40,000 with probability $1/2$. The investor has $u(w) = \sqrt{w}$.
 - (a) (1 point) Compute the expected value of the gamble.
 - (b) (1 point) Compute the expected utility.
2. (2 points) **[Easy] Certainty equivalent and risk premium.** Continue with the previous gamble.
 - (a) (1 point) Compute the certainty equivalent.
 - (b) (1 point) Compute the risk premium. Is the investor risk averse? Why?
3. (3 points) **[Medium] Diversification.** Two independent assets each have an expected return of 8% and a standard deviation of 20%.
 - (a) (1 point) Compute the expected return and standard deviation of a 50/50 portfolio.
 - (b) (1 point) Generalize to n identical, independent, equally weighted assets.
 - (c) (1 point) What if the assets are perfectly correlated ($\rho = 1$)?
4. (3 points) **[Medium] Adverse selection in the used car market.** Half of all cars are good and half are bad. A good car is worth €10,000 to the seller and €12,000 to the buyer; a bad one is worth €4000 and €6000 respectively. Only the seller knows the quality.
 - (a) (1 point) Under perfect information, which transactions take place?
 - (b) (1 point) Under asymmetric information, what is the most a risk neutral buyer will pay? Who agrees to sell?
 - (c) (1 point) Describe the equilibrium and suggest a remedy.
5. (5 points) **[Hard] Demand for insurance and the premium.** A person has wealth of €100,000 and faces a 1% probability of suffering a loss of €40,000. Her utility is $u(w) = \sqrt{w}$.
 - (a) (2 points) Compute expected utility without insurance and the certainty equivalent.
 - (b) (2 points) Find the maximum premium she will pay for full coverage, the actuarially fair premium, and the difference.
 - (c) (1 point) Why can the insurer behave as if it were risk neutral?
6. (5 points) **[Hard] Moral hazard and the deductible.** In the same setting, careful behavior reduces the loss probability from 1% to 0.5%, at an effort cost equivalent to €150. For the incentive analysis, assume risk neutrality.
 - (a) (2 points) With full coverage, does the insured have an incentive to be careful? Would care be socially desirable?
 - (b) (2 points) With a €20,000 deductible, does the answer change? What is the minimum deductible that induces care?
 - (c) (1 point) What does this imply for contract design?