

Principles of Economics and Management I

Ciências ULisboa · MMAEG · 2026/2027

Problem Set 2 · Consumer Choice

Session 2 · LN02

Exercise level: [Easy] routine / warm-up [Medium] typical test level [Hard] challenge (above test level, to consolidate mastery)

1. (2 points) **[Easy] Marginal utilities and their ratio.** A consumer has $U(x, y) = x^{1/2}y^{1/2}$.
 - (a) (1 point) Obtain the marginal utilities U_x and U_y , and their ratio U_x/U_y .
 - (b) (1 point) At the bundle $(x, y) = (4, 9)$, how much y is the consumer willing to give up for one additional unit of x ?
2. (2 points) **[Easy] The budget constraint.** The consumer has income $m = €120$, with $p_x = €4$ and $p_y = €6$.
 - (a) (1 point) Write the budget constraint, its slope and the two intercepts.
 - (b) (1 point) If p_x rises to €6, how does the line change? And if income were to fall to €80 instead?
3. (3 points) **[Medium] The consumer's optimum.** With $U = x^{1/2}y^{1/2}$, $m = €120$, $p_x = €4$ and $p_y = €6$.
 - (a) (1 point) Write the optimality condition and explain its economic meaning.
 - (b) (1 point) Find the optimal bundle (x^*, y^*) .
 - (c) (1 point) Obtain the demand functions for general m , p_x and p_y .
4. (3 points) **[Medium] The Lagrangian and the multiplier.** The same consumer: $U = x^{1/2}y^{1/2}$, $m = €120$, $p_x = €4$, $p_y = €6$.
 - (a) (1 point) Write the Lagrangian and its three first order conditions.
 - (b) (1 point) Solve for λ at the optimum.
 - (c) (1 point) Income rises to €132. Compute the new utility, compare the increase with λ , and say why they nearly agree.
5. (5 points) **[Hard] Marshallian demand and budget shares.** Now the general Cobb-Douglas case, $U(x, y) = x^\alpha y^{1-\alpha}$ with $0 < \alpha < 1$.
 - (a) (2 points) Derive the Marshallian demands $x^*(p_x, p_y, m)$ and $y^*(p_x, p_y, m)$.
 - (b) (1 point) Show that the share of income spent on each good does not depend on prices or on income.
 - (c) (1 point) Show that the demands are homogeneous of degree zero in (p_x, p_y, m) , and say what that means economically.
 - (d) (1 point) A colleague estimates that this household spends 40% of its budget on x when x is cheap and 25% when x is dear. Can their preferences be Cobb-Douglas? Explain.
6. (5 points) **[Hard] A price change, and what it does to λ .** With $U = x^{1/2}y^{1/2}$, $m = €120$ and $p_y = €6$, the price p_x rises from €4 to €6.
 - (a) (1 point) Find the optimal bundle before and after.

- (b) (2 points) Compute λ before and after, and say what its change means.
- (c) (2 points) Compute spending on x before and after the price change, and explain why it comes out as it does.